## Solubility and Precipitates

- Everything dissolves in everything else, but to what extent?
- Rule of thumb: If solubility limit < 0.01 M , it is considered insoluble


## Solubility rules (guidelines)

- All $\mathrm{NO}_{3}^{-}, \mathrm{C}_{2} \mathrm{H}_{3} \mathrm{O}_{2}^{-}, \mathrm{ClO}_{4}^{-}$, Group IA metal ions ( $\mathrm{Li}^{+}, \mathrm{Na}^{+}, \mathrm{K}^{+}, \mathrm{Rb}^{+}, \mathrm{Cs}^{+}$) and $\mathrm{NH}_{4}^{+}$salts are soluble.
- Most $\mathrm{Cl}^{-}, \mathrm{Br}$, and $\mathrm{I}^{-}$salts are soluble.
- Exceptions: $\mathrm{Pb}^{2+}, \mathrm{Ag}^{+}$, and $\mathrm{Hg}_{2}{ }^{2+}$
- Most $\mathrm{SO}_{4}{ }^{2-}$ salts are soluble.
- Exceptions: $\mathrm{Sr}^{2+}, \mathrm{Ba}^{2+}, \mathrm{Pb}^{2+}$ and $\mathrm{Hg}_{2}{ }^{2+}$ ( $\mathrm{Ca}^{2+}$ is slightly soluble)
- Most $\mathrm{CO}_{3}{ }^{2-}, \mathrm{OH}^{-}, \mathrm{PO}_{4}{ }^{3-}$, and $\mathrm{S}^{2-}$ salts are insoluble.
- Exceptions: Group IA metal ions ( $\mathrm{Ca}^{2+}, \mathrm{Ba}^{2+}$ and $\mathrm{Sr}^{2+}$ are slightly soluble)


## If you're not part of the solution...

- You're part of the precipitate!
- In net ionic equations, the precipitate does not dissociate (stays as one entity)
- Example: $\mathrm{AgNO}_{3}+\mathrm{NaCl} \rightarrow \mathrm{AgCl}+\mathrm{NaNO}_{3}$ (overall)
- $\mathrm{Ag}^{+}+\mathrm{NO}_{3}^{-}+\mathrm{Na}^{+}+\mathrm{Cl}^{-} \rightarrow \mathrm{AgCl}+\mathrm{Na}^{+}+\mathrm{NO}_{3}^{-}$(ionic)
- $\mathrm{Ag}^{+}+\mathrm{Cl}^{-} \rightarrow \mathrm{AgCl}$ (net ionic)


## Empirical Gas Laws

- Early experiments conducted to understand the relationship between $P, V$ and $T$ (and number of moles $n$ )
- Results were based purely on observation
- No theoretical understanding of what was taking place
- Systematic variation of one variable while measuring another, keeping the remaining variables fixed


## Boyle's Law (1662)

- For a closed system (i.e. no gas can enter or leave) undergoing an isothermal process (constant T ), there is an inverse relationship between the pressure and volume of a gas (regardless of the identity of the gas)

$$
V \propto 1 / P
$$

- To turn this into an equality, introduce a constant of proportionality (a) $\rightarrow \mathrm{V}=\mathrm{a} / \mathrm{P}$, or $\mathrm{PV}=\mathrm{a}$
- Since the product of PV is equal to a constant, it must UNIVERSALLY (for all values of $P$ and $V$ ) be equal to the same constant

$$
P_{1} V_{1}=P_{2} V_{2}
$$

## Boyle's Law

- There is an inverse relationship between P and V (isothermal, closed system)



## Charles' Law (1787)

- For a closed system undergoing an isobaric process (constant P), there is a direct relationship between the volume and temperature of a gas.

$$
V \alpha T
$$

- Proceeding in a similar fashion as before, we can say that $\mathrm{V}=$ bT (the constant need not the same as that of Boyle's Law!) and therefore $\mathrm{V} / \mathrm{T}=\mathrm{b}$

$$
\frac{V_{1}}{T_{1}}=\frac{V_{2}}{T_{2}}
$$

## Charles' Law (continued)

- This equation presumes a plot of V vs T would give a straight line with a slope of b and a y-intercept of 0 (i.e. equation goes through the origin).
- However the experimental data did have an intercept!
- Lord Kelvin (1848) decided to extrapolate the data to see where it would cross the $x$-axis (T)
- Regardless of the nature of the gas, the data always would yield the same value:
$-273.15^{\circ} \mathrm{C}$
- Therefore to make it go through zero, just add 273.15 to each point!
- This established the Kelvin (absolute) scale


## Absolute temperature (Kelvin)

- The establishment of an absolute temperature scale was based on experiments



## Gay-Lussac's Law (1809)

- For a closed system undergoing an isochoric process (constant V), there is a direct relationship between the pressure and temperature of a gas
- P $\alpha$ T
- Or proceeding as for Charles' Law, $\mathrm{P} / \mathrm{T}=\mathrm{c}$

$$
\frac{P_{1}}{T_{1}}=\frac{P_{2}}{T_{2}}
$$



- In this relation, we also must use absolute temperature


## Avogadro's Law (1811)

- For an open system (mass is allowed to be transferred in or out), the volume of gas is directly proportional to the amount of gas present (given isothermal and isobaric conditions)

$$
\vee \alpha n
$$

- Mathematically this can be written as $V=d n$ (yet another constant)


## Avogadro's Law (continued)

- Since the $T$ and $P$ must be constant, it would be useful to define a reference state ( $T$ and $P$ ) so that gases can be compared to each other
- STP (standard temperature and pressure)
$-\mathrm{T}=0^{\circ} \mathrm{C}$ and $\mathrm{P}=1 \mathrm{~atm}$
- Standard state
$-\mathrm{T}=25^{\circ} \mathrm{C}$ and $\mathrm{P}=1$ bar


## Combined Gas Law (?)

- It would appear that all the relationships can be combined into one equation. For example, volume is seen to be inversely proportional to pressure, directly proportional to the temperature and directly proportional to the number of moles of gas. Therefore

$$
V \alpha n T / P
$$

- Or
- This will eventually ${ }_{1}^{n_{1} T_{1}} n_{n_{2}} T_{2}$ ead to the ideal (perfect) gas law, $\mathrm{PV}=\mathrm{nRT}$, as well.


## Kinetic Molecular Theory

- Provides a theoretical explanation for the behavior of gases
- KMT is simply a model - it is not a perfect description of reality
- Good enough! (if not we will fix it later)


## Postulates of the KMT

- A gas is composed of particles (molecules or atoms) that are perfect spheres
- Gas particles are in constant, random motion
- Gas particles move in straight lines (i.e. not accelerating)
- Gas particles are very far apart
$-V_{\text {gas }} \ll V_{\text {container }}$ (most of a gas is empty space)
- The temperature is proportional to the average kinetic energy of the motion


## More postulates of the KMT

- Gas particles move independently of each other
- The position and momentum of one particle are not affected by the position/momentum of another.
- There are no forces of attraction or repulsion between particles
- Eventually gas particles will collide.
- Collisions with other particles will be perfectly elastic
- Collisions with the walls of the container will result in pressure.

Determine the number of moles of compound and the number of moles of each type of atom in 3.06 $\times 10^{-3} \mathrm{~g}$ of the amino acid glycine, $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{NO}_{2}$.



## Applications of the Ideal Gas Law

- Determination of the molar mass of a gas
- Since PV=nRT we can say that n=PV/RT
- We can also say that the number of moles is given by $n=m / M$
- Setting these expressions equal to each other yields PV/RT = $\mathrm{m} / \mathrm{M}$, which can be rearranged to solve for the molar mass:

$$
M=\frac{m R T}{P V}
$$

## Applications of the Ideal Gas Law

- Determination of the density of a gas
- Let's use the fact that $\rho=m / V$ and rearrange the ideal gas law to yield V=nRT/P
- Combining this with $n=m / M$ and doing some algebra gives

$$
\rho=\frac{M P}{R T}
$$

What is the molar mass of a gas if 0.0494 g of the gas occupies a volume of
0.100 L at a temperature $26^{\circ} \mathrm{C}$ and a pressure of 307 torr?

## What is a mole?

- Small furry animal

- Facial blemish



## What is a mole?

- A mole represent the number of "particles" (elementary entities) present in a sample.
- Avogadro's Number ( $\mathrm{N}_{\mathrm{A}}$ )
$-6.022 \times 10^{23} / \mathrm{mol}$
- The mole can also be related to mass
$-n=m / M$
$-\mathrm{n}=$ \# of moles
$-\mathrm{m}=$ mass
$-\mathrm{M}=$ molar mass (or formula mass or atomic mass)


## Molecular velocities

- The KMT can be used to calculate the root-mean-square velocity

$$
u_{r m s}=\sqrt{\frac{3 R T}{M}}
$$

Maxwell-Boltzmann Distribution of $\mathrm{CO}_{2}$ at Various Temperatures


## Graham's Law of Diffusion/Effusion (1831)

- Diffusion - movement due to a driving force (i.e. concentration gradient)
- Effusion - movement through a small hole
- Both are a "spreading out", and their behavior is based on KMT
- If two gases are at the same temperature, then they must
- Rearranging this, $\frac{u_{2}}{u_{1}}=\sqrt{\frac{M_{1}}{M_{2}}}$

Describe what happens to the average kinetic energy of ideal gas molecules when the conditions are changed as follows:
(a) The pressure of the gas is increased by reducing the volume at constant temperature.
(b) The pressure of the gas is increased by increasing the temperature at constant volume.
(c) The average velocity of the molecules is increased by a factor of 2 .

## Real gases

- There are a number of reasons why the ideal gas law might break down
- Molecular forces
- Attractive and repulsive forces do exist, and may be substantial (i.e. polar molecules)
- Conditions
- Under extreme cases of "high" pressure and/or "low" temperature gases start to behave more like condensed phases (liquids and solids) and intermolecular forces cannot be ignored


## van der Waals equation

- First account for the finite volume that a gas molecule occupies
$-V \rightarrow V$-nb
- Since this will decrease the volume "available" for the molecules to collide, they should collide more often. This will mean that the pressure should increase.
- Mathematically a collision is defined as two particles in the same place at the same time. We can think of the "particle density" as being $n / V$. Thus the number of collision should be proportional to $(n / V)(n / V)=n^{2} / V^{2}$
$-P \rightarrow P+a n^{2} / V^{2}$


## van der Waals equation

- Putting all of this together leads to

$$
\left(P+\frac{a n^{2}}{V^{2}}\right)(V-n b)=n R T
$$

or

$$
\left(P+\frac{a}{v^{2}}\right)(v-b)=R T
$$

where $\mathrm{v}=\mathrm{V} / \mathrm{n} . \mathrm{v}$ is known as the specific volume, or molar volume.
It is important to note that the constants ( $a$ and $b$ ) are different for each substance.

## van der Waals equation

- Typically corrections are "small" but can improve agreement
- v=22.4 L/mol for ideal gas at STP so $b / v$ is $\ll 1$
- Corrections tend to be larger for larger molecules, as well as for polar molecules

| Substance | $a\left(L^{2} \mathrm{~atm} / \mathrm{mol}\right)$ | $b(\mathrm{~L} / \mathrm{mol})$ |
| :--- | :--- | :--- |
| He | 0.0341 | 0.02370 |
| Ar | 1.34 | 0.0322 |
| $\mathrm{H}_{2}$ | 0.244 | 0.0266 |
| $\mathrm{O}_{2}$ | 1.36 | 0.0318 |
| $\mathrm{CO}_{2}$ | 3.59 | 0.0427 |
| $\mathrm{CCl}_{4}$ | 20.4 | 0.1383 |

Under which of the following sets of conditions does a real gas behave most like an ideal gas, and for which conditions is a real gas expected to deviate from ideal behavior? Explain.
(a) high pressure, small volume
(b) high temperature, low pressure
(c) low temperature, high pressure

## A comparison of the three main phases of matter

- Gases, liquids and solids differ from each other in the relative magnitudes of inter- and intramolecular forces

| Phas <br> e | Volume | Shape | Compressibility | Fluidity |
| :--- | :--- | :--- | :--- | :--- |
| Gas | Indefinite | Indefinite | High | High |
| Liqui <br> d | Definite | Indefinite | Low | High |
| Solid | Definite | Definite | Low | Low |

A 2.50-L volume of hydrogen measured at $196{ }^{\circ} \mathrm{C}$ is warmed to 100 ${ }^{\circ} \mathrm{C}$. Calculate the volume of the gas at the higher temperature, assuming no change in pressure.

## Phase diagrams

- Graphical representation of the states of matter as a function of temperature and pressure


From the phase diagram for carbon dioxide, determine the state of $\mathrm{CO}_{2}$ at:
(a) $20^{\circ} \mathrm{C}$ and 1000 kPa
(b) $10^{\circ} \mathrm{C}$ and 2000 kPa
(c) $10^{\circ} \mathrm{C}$ and 100 kPa
(d) $-40^{\circ} \mathrm{C}$ and 500 kPa
(e) $-80^{\circ} \mathrm{C}$ and 1500 kPa (f) $-80^{\circ} \mathrm{C}$ and 10 kPa


## Critical Points

- Critical Temperature $\left(T_{c}\right)$ - highest temperature that liquid and gas can exist as distinct phases
- A liquid can be produced by simply increasing the pressure of the gas
- Critical Pressure $\left(P_{c}\right)$ - highest pressure that liquid and gas can exist as distinct phases
- A liquid can be produced by simply decreasing the temperature of the gas
- Beyond the critical point, supercritical fluid exists


## Critical Point - in pictures

- Phase boundary disappears, so the two phases are indistinguishable



## Triple Point

- For a one component system, there exists a unique temperature and pressure where all three phases coexist at equilibrium
- It is a physical property of the substance and can't be varied!
- Ex. $\mathrm{H}_{2} \mathrm{O} T=0.0098^{\circ} \mathrm{C}$ and P $=4.58 \mathrm{~mm} \mathrm{Hg}$


## Solids

- Most difficult phase to model because particles (ions, atoms, molecules) are in very close contact
- Strongest intermolecular forces
- Amorphous - disorganized clusters with no long-range order
- Crystalline - highly ordered lattice-like assemblies


## Types of Crystalline Solids

- Molecular
- Nonpolar
- Polar
- H-bonded
- Network covalent
- Ionic
- Metallic


## Crystal lattices

- 14 different types
- Lengths (a,b,c)
- Angles ( $\alpha, \beta, \gamma$ )
- Different types of symmetry



## Cubic Arrangements

- Simple cubic cell
- Has its constituents only at the edges (corners) of a cube
- Body-centered cubic (bcc)
- Has an additional constituent at the center of the cube
- Face-centered cubic (fcc)
- Has an additional constituent at the center of each face of the cube


Simple cubic


Body-centered Face-centered cubic cubic

Identify the type of
crystalline solid (metallic, network covalent, ionic, or molecular) formed by each of the following substances:
(a) $\mathrm{SiO}_{2}$
(b) KCl
(c) Cu
(d) $\mathrm{CO}_{2}$
(e) $\mathrm{NH}_{4} \mathrm{~F}$



## Types of solutions

| Solute | Solvent | Solution Phase | Examples |
| :--- | :--- | :--- | :--- |
| Gas | Gas | Gas | Air, natural gas |
| Gas | Liquid | Liquid | Club soda $\left(\mathrm{CO}_{2}\right.$ in $\left.\mathrm{H}_{2} \mathrm{O}\right)$, artificial blood $\left(\mathrm{O}_{2}\right.$ in <br> perfluorodecalin $)$ |
| Liquid | Liquid | Liquid | Vodka |
| Solid | Liquid | Liquid | Saline |
| Gas | Solid | Solid | $\mathrm{H}_{2} / \mathrm{Pd}$ |
| Solid | Solid | Solid | 14 -karat gold $(\mathrm{Ag}$ in Au$)$ |

## Energetics of solution formation

- 1) Pure solvent $\rightarrow$ separated solvent molecules
$-\Delta H_{1}>0$ because intermolecular forces are being broken
- 2) Pure solvent $\rightarrow$ separated solute molecules
$-\Delta \mathrm{H}_{2}>0$ because intermolecular forces are being broken
- 3) Separated solvent and solute molecules $\rightarrow$ solution
$-\Delta \mathrm{H}_{3}<0$ because intermolecular forces are being formed
$\Delta \mathrm{H}_{\text {solution }}=\Delta \mathrm{H}_{1}+\Delta \mathrm{H}_{2}+\Delta \mathrm{H}_{3}$


## What it means to be ideal

- For condensed phases, we know that there are intermolecular forces, which may be fairly significant. Considering the case of just two different types of molecules in a solution ( $A$ and $B$ ), there are really three types of interactions: $\mathrm{A}-\mathrm{A}, \mathrm{A}-\mathrm{B}$ and $\mathrm{B}-\mathrm{B}$.
- If the solution is ideal, then the magnitudes of these interactions are all equal - i.e. it doesn't really matter who your neighbor is!
- $\Delta \mathrm{H}_{\text {solution }}=0, \Delta \mathrm{~V}_{\text {solution }}=0$


## Ways to measure concentration

- Relative/qualitative terms
- Dilute or concentrated
- Solubility - usually based on g of solute / 100 mL of water
- Unsaturated - under solubility limit
- Saturated - at solubility limit
- Supersaturated - over solubility limit


## Ways to measure concentration

- \% by mass

$$
\% \text { by mass }=\frac{\text { mass of solute }}{\text { total mass of solution }} \times 100 \%
$$

- Total mass $=$ mass of solute + mass of solvent
- \% by volume

$$
\% \text { by volume }=\frac{\text { volume of solute }}{\text { total volume of solution }} \times 100 \%
$$

- Total volume $\approx$ volume of solute + volume of solvent


## Ways to measure concentration

- Molarity
- Most common unit of concentration in chemistry

$$
\text { Molarity }=\frac{\text { moles of solute }}{\text { volume of solution }(\text { in } L)}
$$

$-M=n / V$

- Molality

$$
\text { Molality }=\frac{\text { moles of solute }}{\text { mass of solvent }(\text { in } \mathrm{kg})}
$$

$-m=n / m$

- Better because with molarity you are unsure of the actual volume of liquid being added


## Ways to measure concentration

- Normality
- Used almost exclusively for acids and bases
$-N=M^{*} E$, where $M$ is the molarity and $E$ is the \# of equivalents (\# of $\mathrm{H}^{+}$that will dissociate in an acid, or \# of $\mathrm{OH}^{-}$that will dissociate in a base)
- Useful for titrations
- Mole fraction

$$
X_{A}=\frac{n_{A}}{n_{\text {tot }}}=\frac{n_{A}}{\sum n_{i}} .
$$

- Useful for colligative properties, chemical processes

> Determine the molarity when 98.0 g of phosphoric acid, $\mathrm{H}_{3} \mathrm{PO} 4$, is dissolved in 1.00 L of solution.



## Raoult's Law

- Consider a solution made up of solvent A (large purple spheres) and solute B (small green spheres).
- The rate at which A leaves the surface (vaporization) is proportional to how many you have on the surface, which is proportional to the mole fraction: $\mathrm{r}=\mathrm{kx}_{\mathrm{A}}$
- The rate at which A comes back (condensation) is proportional to the concentration of the gas, which is proportional to the partial pressure: $r=k^{\prime} P_{A}$
- Since these two rates must be the same:

$$
P_{A}=\frac{k}{k^{\prime} x_{A}} .
$$

For a pure liquid $x_{A}=1$ so $k / k^{\prime}=P_{A}$. This means that


$$
P_{A}=x_{A} P_{A}^{*}
$$

## Ideal solutions

- An ideal solution is one where Raoult's Law is obeyed.
- Since $P=P_{A}+P_{B}$, and the vapor pressure for a liquid is the same as that for a gas, for an ideal solution we carp saypthix $+P_{B}{ }^{*} x_{B}$
- Furthermore, since $x_{A}+x_{B}=1$,

$$
\begin{aligned}
& P=P_{A}^{*} x_{A}+P_{B}^{*}\left(1-x_{A}\right) \text { or } \\
& P=\left(P_{A}^{*}-P_{B}^{*}\right) x_{A}+P_{B}^{*}
\end{aligned}
$$

## Example of an ideal solution

- Solutions tend to behave ideally when the solvent ( $A$ ) and solute ( $B$ ) are "similar" to each other in terms of molecular structure, polarity, intermolecular forces, etc.
- Ex. Benzene and toluene (methylbenzene)





## Colligative properties

- The presence of a solute can affect the properties of a solution
- This effect is (primarily) due to the amount of solute (i.e. concentration) but not necessarily on the nature of the solute
- Three main colligative properties:
- Boiling point elevation
- Freezing point depression
- Osmotic pressure


## Freezing point depression and boiling point elevation

- Consider solutions where only the solvent is volatile, and the solute only dissolves in the liquid phase of the solvent

- $\Delta \mathrm{T}_{\mathrm{f}}=-\mathrm{i} \mathrm{K}_{\mathrm{f}} m$
- $\Delta \mathrm{T}_{\mathrm{b}}=\mathrm{iK} \mathrm{b}_{\mathrm{b}} m$
$\mathrm{i}=$ van't Hoff factor ( $\mathrm{i}=1$ for a nonelectrolyte, $\approx 1$ for a weak electrolyte, or \# of particles for a strong electrolyte)
$\mathrm{K}_{\mathrm{f}}=$ freezing-point depression (cryoscopic) constant $\mathrm{K}_{\mathrm{b}}=$ boiling-point elevation (ebullioscopic) constant
$m=$ molality


## Osmotic pressure

- Osmosis - net flow of solvent molecules through a semipermeable membrane
- Solvent molecules go from a solution of lower concentration to a solution of higher concentration (solute is not able to pass through)
- Osmotic pressure $(\pi)=$ pressure required to stop osmosis

- $\pi=$ MRT, where $\mathrm{M}=$ molarity

